1. **Formal power series**. Define the function $E(x)$ as a formal power series as follows:
$$E(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$$
Show that $E(x+y) = E(x) \cdot E(y)$.

2. **Finding a generating function from a recurrence**. The *Lucas numbers* are defined by the recurrence relation
$$L_n = L_{n-1} + L_{n-2}$$
where $L_0 = 2$ and $L_1 = 1$. Find the generating function for the Lucas numbers and use it to find an explicit formula for the nth Lucas number.

3. **Finding a generating function**. Find a generating function for
$$A(n) = \binom{n}{2}$$

4. **Multiplying generating functions**. n distinct people are about to get on an airplane from Los Angeles to Chicago; some subset of the passengers are in first class, and the rest are in economy. Everyone is lined up in the jetway, and since first-class flyers get priority boarding, they are all before the economy-class flyers.

   The first class passengers all have their choice of two possible dinners; steak or fish. Out of the economy class passengers, two (we don't know which ones) have pre-ordered meals (they get lasagna). All the rest get nothing.

   Given the line of people waiting to get on, how many ways are there for the passengers to get meals? Find a generating function and solve for a general form.

   How would the problem change if the passengers were still waiting at the gate and hadn't lined up yet?