1. **Graph automorphisms**

   (a) Define, in your own words, what a graph automorphism is.

   (b) Let G be the graph of a cube. (That is, the vertices of G are the corners of the cube and the edges of G follow the edges of the cube). G has 8 vertices and 12 edges. How many automorphisms does G have?

   (c) Suppose we take G and draw in a 'long' diagonal, that is, a diagonal connecting two opposite corners of the cube. How many automorphisms does this graph have?

   (d) Suppose that instead of a 'long' diagonal we draw a 'short' diagonal, connecting two corners of the same face of the cube. How many automorphisms does this graph have?

2. **Counting permutations with certain types of cycle structures**.

   (a) How many permutations of $[n]$ are there whose cycle structure consists entirely of k-cycles?

   (b) Find an exponential generating function for your answer in part (a).

   (c) A permutation σ is called an *involution* if $\sigma^2 = \mathrm{Id}$. Find an exponential generating function for the number of involutions of $[n]$.

   (d) (If you have extra time). Find an exponential generating function for the number of permutations whose fourth power is the identity. What is the seventh derivative of this function evaluated at zero?

   (e) (Mathematical alchemy). Using the previous results, find an exponential generating function for the number of permutations with *any* cycle structure. Simplify using a Taylor series. Does the result make sense?