Counting

1. In my 31B class last quarter, there were 151 students. How many ways could those students be partitioned among the three TAs if

   - the students are all the same (so we just care about the number of students each TA gets)?
   - the students are all different (so we care about the exact roster that each TA gets)?

   Show that at least one TA got more than 50 students.

2. Let X be a set with n elements, and let $0 \leq k \leq n$. Then define a to be the number of subsets of x of size k, and b to be the number of injective functions $[k] \to X$. Use the division principle to show

$$b = k! \cdot a.$$

   Verify the above equality using the formulas for a and b.

3. For any positive integer n,

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n.$$

   Prove this fact by induction on n. Then use the combinatorial interpretation of binomial coefficients to justify this equality combinatorially. (There is another, very slick proof of this result. Can you find it?)

   Challenge

4. Suppose we have a single-elimination tournament with 2^n participants. That is, there are n rounds; in the first, every team plays one other team, and the losing teams are eliminated (there are no ties). Then the remaining teams play one another, and this continues until there is a single winner. The question is, how many different ways are there to set up such a tournament?

   If $n = 1$ there is only one possibility. If $n = 2$ there are three possible tournaments.

   - Show that if $n = 3$ there are 315 ways to set up the tournament.
   - Find a general formula for the number of possible setups in terms of n.
   - Show that the number of setups is always odd.