1. **Generating function**. Find an exponential generating function for the number of permutations of $[n]$. Can you find an ordinary generating function for this sequence?

2. **Fixed points**. Let σ be a permutation of $[n]$, that is, a bijection $[n] \to [n]$. We call $k \in n$ a *fixed point* of σ if $\sigma(k) = k$. Earlier, we counted the permutations with no fixed points (these are the derangements). Find formulas for the number of permutations with n, $n-1$, $n-2$, and $n-3$ fixed points.

   Find a general formula for the number of permutations of $[n]$ with k fixed points.

   (If you know some probability: Use the fact that the number of derangements of $[n]$ is approximately $n!/e$ to show that, for large n, the number of fixed points of a random permutation is approximately Poisson-distributed with rate 1).

3. **Small permutations**. Consider the set of all permutations on $[4]$. There are five different types of cycle structure for the permutations in this set:

   - Four 1-cycles
   - A 2-cycle and two 1-cycles
   - Two 2-cycles
   - A 3-cycle and a 1-cycle
   - A 4-cycle

   How many of each of these are there? Your answers should add up to $4! = 24$.

   (If you have extra time, try doing the same thing for permutations of 5 elements).